

PHASE CONVERSION AFTER A CHIRAL TRANSITION: EFFECTS FROM INHOMOGENEITIES AND FINITE SIZE*

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We discuss the kinetics of phase conversion, through the nucleation of bubbles and spinodal decomposition, after a chiral transition within an effective field theory approach to low-energy QCD. We study possible effects resulting from the finite size of the expanding system for both the initial and the late-stage growth of domains, as well as those effects due to inhomogeneities in the chiral field which act as a background for the fermionic motion.

To model the mechanism of chiral symmetry breaking present in QCD, and to study the dynamics of phase conversion after a temperature-driven chiral transition, one can resort to low-energy effective models. In particular, to study the mechanisms of bubble nucleation and spinodal decomposition in a hot expanding plasma, it is common to adopt the linear σ -model coupled to quarks, where the latter comprise the hydrodynamic degrees of freedom of the system^{1,2,3,4,5,6}. The gas of quarks provides a thermal bath in which the long-wavelength modes of the chiral field evolve, and the latter plays the role of an order parameter in a Landau-Ginzburg approach to the description of the chiral phase transition. The gas of quarks and anti-quarks is usually treated as a heat bath for the chiral field, with temperature T . The standard procedure is then integrating over the fermionic degrees of freedom, using a classical approximation for the chiral field, to obtain a formal expression for the thermodynamic potential of an infinite system.

Let us consider a chiral field $\phi = (\sigma, \vec{\pi})$, where σ is a scalar field and π^i are pseudoscalar fields playing the role of the pions, coupled to two flavors

*Presented at Strong and Electroweak Matter 2004, Helsinki, June 16-19, 2004

of quarks according to the Lagrangian

$$\mathcal{L} = \bar{q}[i\gamma^\mu\partial_\mu + \mu_q\gamma^0 - M(\phi)]q + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) . \quad (1)$$

Here q is the constituent-quark field $q = (u, d)$ and $\mu_q = \mu/3$ is the quark chemical potential. The interaction between the quarks and the chiral field is given by $M(\phi) = g(\sigma + i\gamma_5\vec{\tau} \cdot \vec{\pi})$, and $V(\phi) = \frac{\lambda^2}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - h_q\sigma$ is the self-interaction potential for ϕ .

The parameters above are chosen such that chiral $SU_L(2) \otimes SU_R(2)$ symmetry is spontaneously broken in the vacuum. The vacuum expectation values of the condensates are $\langle\sigma\rangle = f_\pi$ and $\langle\vec{\pi}\rangle = 0$, where $f_\pi = 93$ MeV is the pion decay constant. The explicit symmetry breaking term is due to the finite current-quark masses and is determined by the PCAC relation, giving $h_q = f_\pi m_\pi^2$, where $m_\pi = 138$ MeV is the pion mass. This yields $v^2 = f_\pi^2 - m_\pi^2/\lambda^2$. The value of $\lambda^2 = 20$ leads to a σ -mass, $m_\sigma^2 = 2\lambda^2 f_\pi^2 + m_\pi^2$, equal to 600 MeV. For $g > 0$, the finite-temperature one-loop effective potential also includes a contribution from the quark fermionic determinant. In what follows, we treat the gas of quarks as a heat bath for the chiral field, with temperature T and baryon-chemical potential μ . Then, one can integrate over the fermionic degrees of freedom, obtaining an effective theory for the chiral field ϕ . Using a classical approximation for the chiral field, one obtains the thermodynamic potential

$$\Omega(T, \mu, \phi) = V(\phi) - \frac{T}{\mathcal{V}} \ln \det\{[G_E^{-1} + W(\phi)]/T\} , \quad (2)$$

where G_E is the fermionic Euclidean propagator. From the thermodynamic potential one can obtain all the thermodynamic quantities of interest.

To compute correlation functions and thermodynamic quantities, one has to evaluate the fermionic determinant within some approximation scheme. In $1D$ systems one can usually resort to exact analytical methods⁷. In practice, however, the determinant is usually calculated to one-loop order assuming a homogeneous and static background chiral field. Nevertheless, for a system that is in the process of phase conversion after a chiral transition, one expects inhomogeneities in the chiral field to play a role in driving the system to the true ground state.

We propose an approximation procedure to evaluate the finite-temperature fermionic determinant in the presence of a chiral background field, which systematically incorporates effects from inhomogeneities in the chiral field through a derivative expansion. The method is valid for the case in which the chiral field varies smoothly, and allows one to extract infor-

mation from its long-wavelength behavior, incorporating corrections order by order in the derivatives of the field.

The Euler-Lagrange equation for static chiral field configurations contains a term which represents the fermionic density $\rho(\vec{x}_0) = (\nu_q/\mathcal{V}) \langle \vec{x}_0 | (G_E^{-1} + M(\hat{x}))^{-1} | \vec{x}_0 \rangle$, where $|\vec{x}_0\rangle$ is a position eigenstate with eigenvalue \vec{x}_0 , and $\nu_q = 12$ is the color-spin-isospin degeneracy factor. In momentum representation:

$$\rho(\vec{x}_0) = \nu_q T \sum_n \int \frac{d^3 k}{(2\pi)^3} e^{-i\vec{k} \cdot \vec{x}} \frac{1}{\gamma^0(i\omega_n + \mu) - \vec{\gamma} \cdot \vec{k} + M(\hat{x})} e^{i\vec{k} \cdot \vec{x}}. \quad (3)$$

We can transfer the \vec{x}_0 dependence to $M(\hat{x})$ through a unitary transformation, expand $M(\hat{x} + \vec{x}_0)$ around \vec{x}_0 , and use $\hat{x}^i = -i\nabla_{k_i}$ to write

$$\rho(\vec{x}_0) = \nu_q T \sum_n \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\gamma^0(i\omega_n + \mu) - \vec{\gamma} \cdot \vec{k} + M(\vec{x}_0)} \times \left[1 + \Delta M(-i\nabla_{k_i}, \vec{x}_0) \frac{1}{\gamma^0(i\omega_n + \mu) - \vec{\gamma} \cdot \vec{k} + M(\vec{x}_0)} \right]^{-1}, \quad (4)$$

where \vec{x}_0 is a c-number, not an operator, and $\Delta M(\hat{x}, \vec{x}_0) = \nabla_i M(\vec{x}_0) \hat{x}^i + \frac{1}{2} \nabla_i \nabla_j M(\vec{x}_0) \hat{x}^i \hat{x}^j + \dots$.

If we focus on the long-wavelength properties of the chiral field and assume that the static background, $M(\vec{x})$, varies smoothly and fermions transfer a small ammount of momentum to the chiral field, we can expand the expression above in a power series:

$$\rho(\vec{x}) = \nu_q T \sum_n \int \frac{d^3 k}{(2\pi)^3} \frac{1}{\gamma^0(i\omega_n + \mu) - \vec{\gamma} \cdot \vec{k} + M(\vec{x})} \times \sum_{\ell} (-1)^{\ell} \left[\Delta M(-i\nabla_{k_i}, \vec{x}) \frac{1}{\gamma^0(i\omega_n + \mu) - \vec{\gamma} \cdot \vec{k} + M(\vec{x})} \right]^{\ell}, \quad (5)$$

$$\Delta M(-i\nabla_{k_i}, \vec{x}) = \nabla_i M \left(\frac{1}{i} \right) \nabla_{k_i} + \frac{1}{2} \nabla_i \nabla_j M \left(\frac{1}{i} \right)^2 \nabla_{k_i} \nabla_{k_j} + \dots, \quad (6)$$

which provides a systematic procedure to incorporate corrections brought about by inhomogeneities in the chiral field to the quark density, so that we can calculate $\rho(\vec{x}) = \rho_0(\vec{x}) + \rho_1(\vec{x}) + \rho_2(\vec{x}) + \dots$ order by order in powers of the derivative of the background, $M(\vec{x})$. The leading-order term in this gradient expansion for $\rho(\vec{x})$ can be easily calculated and yields the well-known mean field result for ρ . The net effect of this leading term is to

correct the potential for the chiral field to $V_{eff} = V(\phi) + V_q(\phi)$, where

$$V_q \equiv -\nu_q T \int \frac{d^3 k}{(2\pi)^3} \ln \left(e^{[E_k(\phi) - \mu_q]/T} + 1 \right) + (\mu_q \rightarrow -\mu_q), \quad (7)$$

where $E_k(\phi) = \sqrt{\vec{k}^2 + M(\phi)}$. This sort of effective potential is commonly used as the thermodynamic potential in a phenomenological description of the chiral transition for an expanding quark-gluon plasma created in a high-energy heavy-ion collision^{2,3,4,5,6}. However, the presence of a non-trivial background field configuration, *e.g.* a bubble, can in principle dramatically modify the Dirac spectrum, hence the determinant^{7,8}. Results for the correction of the Laplacian term will be presented elsewhere⁹.

In the process of phase conversion through bubble nucleation in a QGP of finite size, the set of all supercritical bubbles integrated over time will eventually drive the entire system to its true vacuum. The scales that determine the importance of finite-size effects are the typical linear size of the system, the radius of the critical bubble and the correlation length. For definiteness, let us assume our system is described by a coarse-grained Landau-Ginzburg potential, $U(\phi, T)$, whose coefficients depend on the temperature. For the case to be considered, the scalar order parameter, ϕ , is *not* a conserved quantity, and its evolution is given by the time-dependent Landau-Ginzburg equation

$$\frac{\partial \phi}{\partial t} = \gamma [\nabla^2 \phi - U'(\phi, T)] \quad , \quad (8)$$

where γ is the response coefficient which defines a time scale for the system. The equation above is a standard reaction-diffusion equation, and describes the approach to equilibrium.

If $U(\phi, T)$ is such that it allows for the existence of bubble solutions (taken to be spherical for simplicity), then supercritical (subcritical) bubbles expand (shrink), in the thin-wall limit, with the following velocity:

$$\frac{dR}{dt} = \gamma(d-1) \left[\frac{1}{R_c} - \frac{1}{R(t)} \right] \quad , \quad (9)$$

where $R_c = (d-1)\sigma/\Delta F$ and ΔF is the difference in free energy between the two phases. The equation above relates the velocity of a domain wall to the local curvature. The response coefficient, γ , can be related to some characteristic collision time. One can measure the importance of finite-size effects for the case of heavy-ion collisions by comparing, for instance, the asymptotic growth velocity ($R \gg R_c$) for nucleated hadronic bubbles to the expansion velocity of the plasma. In the Bjorken picture, the typical

length scale of the expanding system is $L(T) \approx (v_z t_c)(T_c/T)^3 = L_0(T_c/T)^3$, where v_z is the collective fluid velocity and $L_0 \equiv L(T_c)$ is the initial linear scale of the system for the nucleation process which starts at $T \leq T_c$.

The relation between time and temperature provided by the cooling law that emerges from the Bjorken picture suggests the comparison between the following “velocities”:

$$v_b \equiv \frac{dR}{dT} = - \left(\frac{3b\ell L_0}{2v_z \sigma T_c^2} \right) \left(\frac{T_c}{T} \right)^5 \left(1 - \frac{T}{T_c} \right) \quad , \quad (10)$$

the asymptotic bubble growth “velocity”, and the plasma expansion “velocity” $v_L \equiv (dL/dT) = -(3L_0/T_c)(T_c/T)^4$. The quantity b is a number of order one to first approximation, and comes about in the estimate of the phenomenological response coefficient $\gamma(T) \approx b/2T$. Using the numerical values adopted previously and $\sigma/T_c^3 \sim 0.1$, we obtain¹⁰

$$\frac{v_b}{v_L} \approx \frac{20}{v_z} \left(\frac{T_c}{T} - 1 \right) \quad . \quad (11)$$

One thus observes that the bubble growth velocity becomes larger than the expansion velocity for a supercooling of order $\theta \approx v_z/20 \leq 5\%$. A simple estimate points to a critical radius larger than 1 fm at such values of supercooling⁴. Therefore, finite-size effects appear to be an important ingredient in the phase conversion process right from the start in the case of high-energy heavy-ion collisions¹⁰.

Acknowledgments

Part of this paper is based on work done in collaboration with R. Venugopalan. E.S.F. is partially supported by CAPES, CNPq, FAPERJ and FUJB/UFRJ.

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